ŞEPBEŞIK KONWEKSIÝALY SUWUKLYGYŇ ERKIN YRGYLDYLARY BARADAKY MESELE DOGRUSYNDA

Allaberdy Aşirow
Türkmenistanyň Ylymlar akademiýasynyň prezidenti

Gysgaça beýan

Bu makalada maýyşgak gapdaky şepbeşik gysylmaýan, konweksiýaly suwuklygyň erkin yrgyldylarynyň synag-derňew işleri alnyp baryldy. Degişli spektral meseläniň hususy bahalarynyň strukturasy, olaryň ýerleşişi we operatorlar dessesiniň hususy wektorlarynyň dolulygy hem-de minimallygy baradaky teoremalar subut edildi.

Esasy sözler: maýyşgak gap, spektral mesele, hususy baha, hususy funksiýalar, gysylmaýan suwuklyk, şepbeşik suwuklyk, konweksiýa, funksional giňişlikler, operatorlar, operatorlar dessesi, dolulyk.

GIRIŞ

Türkmenistanyň çuňňur hormatlanylýan Prezidentiniň parasatly ýolbaşçylygy we ýadawsyz zähmeti hem-de türkmen halkynyň Milli Lideri, Gahryman Arkadagymyzyň taýsyz tagallalarynyň netijesinde Berkarar döwletiň täze eýýamynyň Galkynyşy döwründe ýurduň ylym ulgamy häzirki zaman innowasion tehnologiýalaryň önümçilige ornaşdyrylmagy bilen tiz ösýän esasy pudaklaryň birine öwrüldi.

Suwuklyk bilen doly ýa-da bölek doldurylan jisimleriň dinamikasy baradaky meseleleri derňemek, XX asyryň ikinji ýarymyndan soň ýurdumyzyň we daşary ýurt alymlarynyň ünsüni has hem özüne çekip gelipdir. Bu meselelere garamak diňe bir amaly taýdan zerurlygy bilen ähmiýetli bolman, eýsem matematiki nukdaýnazardan hem çuňňur gyzyklanma döredýär. Çünki şeýle meselelerde ýüze çykýan gyra we spektral meseleler diýseň özboluşlydyr. Şunlukda çyzykly meseleler üçin esas düzüji bolup, meseläniň spektral häsiýetleri, ýagny çözüwiň wagta eksponensial kanun boýunça bagly häsiýetleri hyzmat edýär. Spektral meseleler özbaşdak derňelmegi talap edýär. Şunlukda amaly taýdan zerurlygy üçin gyra şertlerde spektral parametri özünde saklaýan, ýagny wagta görä önüm diňe bir deňlemelerde däl-de, eýsem, gyra şertleriň düzümine hem girýän meseleler has hem wajypdyr.

Meseläniň goýluşy. $\boldsymbol{R}^3 = \{(x_1, x_2, x_3)\}$ giňişlikde $\Omega_0 \subset \Omega_1$ bolýan çäkli Ω_0 we Ω_1 oblastlara garalyň we $\Omega = \Omega_1 \setminus \Omega_0$ belgileme girizeliň.

Goý, Ω oblasty eýeleýän maýyşgak jisimde Ω_0 oblast şepbeşik, gysylmaýan suwuklyk bilen doldurylan bolup, $\Sigma = \partial\Omega_0$ ýagny Ω_0 oblastyň çägi, $\Sigma_1 = \partial\Omega \setminus \Sigma$ bolsa Ω oblastyň çäginiň daşky tarapy (bölegi) bolsun. $\Sigma, \Sigma_1 \in \boldsymbol{C}^2$ diýip hasap edeliň.

Maýyşgak jisimde ýylylyk prosesleri geçmeýär, emma suwuklykda konweksiýa bar diýip hasap edeliň.

$\boldsymbol{u}(\boldsymbol{x},t) = (u_1, u_2, u_3)$ bilen $\boldsymbol{x} \in \Omega$ nokadyň t wagt pursadyndaky gyşarmasyny, $\boldsymbol{V}(\boldsymbol{x},t) = (\vartheta_1, \vartheta_2, \vartheta_3)$ bilen suwuklygyň bölejikleriniň tizlik meýdanyny belgiläliň.

Goý, $\rho(\boldsymbol{x})$ maýyşgak jisimiň dykyzlygy, $P(\boldsymbol{x},t)$ bolsa suwuklygyň basyşynyň üýtgemesi bolsun. $T(\boldsymbol{x},t)$ bilen suwuklygyň bölejikleriniň tempiraturasynyň üýtgemegini belgiläliň. Maýyşgak jisim-izotrop diýip hasap ederis. Bu ýagdaýda jisimiň tenzor güýjenmesi

$$\sigma_{jk}(\boldsymbol{u}) = \lambda_0 \delta_{jk} di\vartheta\, \boldsymbol{u} + \mu_0 \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right)$$

görnüşde bolar.

$(j, k = 1,2,3)$ δ_{jk} – Kronekeriň simwoly λ_0 we μ_0 bolsa Lýame hemişelikleridir. Goý,

$$(L\boldsymbol{u})_j = -\sum_{k=1}^{3} \frac{\partial \sigma_{jk}(\boldsymbol{u})}{\partial x_k} \qquad j = 1,2,3$$

bolsun. Onda bu mehaniki sistemanyň kiçi hereketlerini derňemek üçin aşakdaky sistemany alarys (A. Aşirow, 2022).

$$\begin{cases} -\sum_{k=1}^{3} \dfrac{\partial \sigma_{jk}(\boldsymbol{u})}{\partial x_k} + \rho \dfrac{\partial^2 u_j}{\partial t^2} = 0, \ (\Omega); \ j = 1,2,3; \quad \sigma(\boldsymbol{u})\boldsymbol{n}\big|_{£'} = 0. & (1) \\ \nu\Delta \boldsymbol{V} - \nabla P - \dfrac{\partial \boldsymbol{V}}{\partial t} + \boldsymbol{k}T = 0, \ div\boldsymbol{V} = 0 \quad (\Omega_0) & (2) \\ -\Delta T - \boldsymbol{k}\boldsymbol{V} + \dfrac{\partial T}{\partial t} = 0 \quad (\Omega_0) & (3) \\ \sigma(\boldsymbol{u})\boldsymbol{n}\big|_{\Sigma} = -T(\boldsymbol{V})\boldsymbol{n}\big|_{\Sigma}, \ \boldsymbol{u} = \boldsymbol{V}, \quad (\Sigma) & (4) \\ T(\boldsymbol{x},t) = 0, \quad (\Sigma') & (5) \end{cases}$$

Bu ýerde

$$T(\boldsymbol{V}) = -\delta_{jk}P + \nu\left(\frac{\partial \vartheta_j}{\partial x_k} + \frac{\partial \vartheta_k}{\partial x_j}\right) -$$

ululyk $(\boldsymbol{V}, P)$ akyma degişli tenzor güýjenmesi;

ν – şepbeşikligiň kinematik koeffisiýenti;

$\boldsymbol{k}$ – wektor x_3 okunyň orty;

$\boldsymbol{n}$ – daşky normal.

(4) şertler, degişlilikde, Σ üstde güýjenmeleriň we tizlikleriň deňligini aňladýar.

Bu sistemanyň çözüwini

$$\boldsymbol{u}(\boldsymbol{x},t) = exp(-\lambda t)\boldsymbol{u}(\boldsymbol{x}); \quad \boldsymbol{V}(\boldsymbol{x},t) = exp(-\lambda t)\boldsymbol{V}(\boldsymbol{x});$$

$$T(\boldsymbol{x},t) = exp(-\lambda t)T(\boldsymbol{x}); \ P(\boldsymbol{x},t) = exp(-\lambda t)P(\boldsymbol{x}).$$

görnüşde gözläliň. Onda λ – spektral parametre görä

$$\begin{cases} \sum_{k=1}^{3} \dfrac{\partial \sigma_{jk}}{\partial x_k} - \lambda^2 \rho u_j = 0, \ (\Omega); \quad j = 1,2,3; \ \sigma(\boldsymbol{u})\boldsymbol{n}\big|_{£'} = 0. & (6) \\ -\nu\Delta \boldsymbol{V} + \nabla P - \lambda \boldsymbol{V} - \boldsymbol{k}T = 0; \ di\vartheta\boldsymbol{V} = 0 \quad (\Omega_0). & (7) \\ -\Delta T - \boldsymbol{k}\boldsymbol{V} - \lambda \boldsymbol{T} = 0 \quad (\Omega_0). & (8) \\ T(\boldsymbol{x}) = 0; \quad (\Sigma'). & (9) \\ \sigma(\boldsymbol{u})\boldsymbol{n}\big|_{\Sigma} = -T(\boldsymbol{V})\boldsymbol{n}\big|_{\Sigma}; \quad \boldsymbol{u} = \boldsymbol{V} \quad (\Sigma). & (10) \end{cases}$$

meseläni alarys. Hut şu sistema derňewiň obýekti bolup, bu spektral meseläniň hususy bahalaryny we hususy funksiýalaryny derňemek maksat edinilýär.

Funksional giňişliklere we kömekçi meselelere seredeliň.

Ω_0 oblastda kwadraty bilen jemlenýän üçkomponentli wektor-funksiýalaryň giňişligini $\boldsymbol{L}_2(\Omega_0)$ bilen belgiläliň. $\boldsymbol{W}_2^k(\Omega_0)$ bolsa Ω_0 oblastda S.L.Sobolewiň giňişligi bolsun. Edil şuňa meňzeşlikde $\boldsymbol{L}_2(\Sigma)$ we $\boldsymbol{W}_2^k(\Sigma)$ giňişlikler kesgitlenilýär.

$$\Im(\Omega_0) = \{\boldsymbol{V}(x) \epsilon \boldsymbol{L}_2(\Omega_0) : div\boldsymbol{V} = 0\} \qquad (\Omega_0)$$

$$\widehat{\boldsymbol{W}}{}_2^1(\Omega_0) = \boldsymbol{W}_2^1(\Omega_0) \cap \Im(\Omega_0)$$

belgilemelerini girizeliň. $\widehat{\boldsymbol{W}}{}_2^1(\Omega_0)$ giňişligiň $\boldsymbol{W}_2^1(\Omega_0)$ giňişligiň bölek giňişligi bolýandygy düşnüklidir.

Aşakdaky kömekçi meselelere garalyň.

1-nji mesele. $f \in \boldsymbol{L}_2(\Omega_0)$ funksiýasy boýunça

$$L\boldsymbol{u}_1 + \boldsymbol{u}_1 = f; \qquad (\Omega), \quad \sigma(\boldsymbol{u}_1)\boldsymbol{n}\big|_{\partial\Omega} = 0.$$

2-nji mesele. $\varphi \in \boldsymbol{L}_2(\Omega_0)$ funksiýasy boýunça

$$L\boldsymbol{u}_2 + \boldsymbol{u}_2 = 0; \qquad \sigma(\boldsymbol{u}_2)\boldsymbol{n}\big|_{\Sigma_1} = 0; \qquad \sigma(\boldsymbol{u}_2)\boldsymbol{n}\big|_{\Sigma} = \varphi.$$

3-nji mesele. $g \in I(\Omega_0)$ funksiýasy boýunça

$$\nu\mathcal{L}\boldsymbol{V}_1 + \boldsymbol{V}_1 + \nabla P_1 = g; \quad (\Omega_0); \quad T(\boldsymbol{V}_1)\boldsymbol{n}\big|_{\Sigma} = 0$$

meseläniň $\boldsymbol{V}$, $\nabla P \in I(\Omega_0)$ çözüwini tapmaly. Bu ýerde

$$\mathcal{L} = -P\Delta, \quad P : \boldsymbol{L}_2(\Omega_0) \to \Im(\Omega_0)$$

ortoproýektor.

4-nji mesele. $\psi \in \boldsymbol{L}_2(\Sigma)$ funksiýasy boýunça

$$\nu\mathcal{L}\boldsymbol{V}_2 + \boldsymbol{V}_2 + \nabla P_2 = 0; \quad (\Omega_0); \quad T(\boldsymbol{V}_2)\boldsymbol{n}\big|_{\Sigma} = \psi$$

meseläniň $\boldsymbol{V}_2, \nabla P_2 \in \Im(\Omega_0)$ çözüwini tapmaly.

5-nji mesele. $\theta \in \boldsymbol{L}_2(\Omega_0)$ funksiýasy boýunça

$$-\Delta T = \theta, \quad (\Omega_0); \quad T = 0, \quad (\Sigma)$$

meseläniň çözüwini tapmaly.

1-nji lemma. $\boldsymbol{W}_2^1(\Omega_0)$ giňişlikde 1-nji meseläniň ýeke-täk umumylaşdyrylan çözüwi bar bolup, meselede $\boldsymbol{u}_1 = A^{-1}f$, $A^{-1} > 0$

$$A^{-1} : \boldsymbol{L}_2(\Omega_0) \to \boldsymbol{L}_2(\Omega_0); \qquad \mathfrak{D}(A^{1/2}) = \boldsymbol{W}_2^1(\Omega_0)$$

bolýan kompakt, öz-özüne çatrymly, položitel operator emele gelýär.

2-nji lemma. $\boldsymbol{W}_2^1(\Omega_0)$ giňişlikde 2-nji meseläniň $\forall\varphi \in \boldsymbol{L}_2(\Omega_0)$ üçin ýeke-täk umumylaşdyrylan çözüwi bar bolup, bu meselede çyzykly, çäkli A_1 operator ýüze çykýar.

Bu lemmalaryň subudy (A. Aşirow, 2023) işde görkezilýär.

(S.G. Kreýn, 1968) işiň usullary bilen 3-nji, 4-nji, 5-nji meseleleriň ýeke-täk çözüwleriniň bardygyny we ol meselelerde degişli operatorlar ýüze çykyp, bu meseleleriň çözüwini kesgitleýändigini görkezmek bolar. Bu operatorlary yzygiderlikde kesgitläp, olaryň käbir häsiýetlerini belläp geçeliň.

3-nji meselede položitel $A_2^{-1} : \Im(\Omega_0) \to \Im(\Omega_0)$ operatory emele gelýär. Şunlukda, $\mathfrak{D}(A_2^{1/2}) = \boldsymbol{W}_2^1(\Omega_0)$ we $\boldsymbol{V} = A_2^{-1}g$.

4-nji meselede $A_3 \boldsymbol{W}_2^{-1/2}(\Sigma) \to \widehat{\boldsymbol{W}}_2^1(\Omega_0)$ operatory emele gelip $\boldsymbol{V} = A_3\psi$ düzgün boýunça täsir edýär.

5-nji meselede $A_T^{-1} : \boldsymbol{L}_2(\Omega_0) \to \boldsymbol{L}_2(\Omega_0)$ operatory emele gelýär. Şunlukda, $\mathfrak{D}\left(A_T^{1/2}\right) = \boldsymbol{W}_2^1(\Omega_0)$ we $T = A_T^{-1}\theta$ bolar.

Girizilen operatorlaryň kömegi bilen (6)-(10) meseleler aşakdaky görnüşde ýazylyp bilner:

$$\begin{cases} \boldsymbol{u} = -\lambda^2 \rho A^{-1}\boldsymbol{u} + A^{-1}\boldsymbol{u} + A_1\left(\sigma(\boldsymbol{u})\boldsymbol{n}\right)_\Sigma; \\ \boldsymbol{V} = (\lambda+1)A_2^{-1}\boldsymbol{V} + A_2^{-1}\overline{P}\boldsymbol{k}T + A_3(T(\boldsymbol{V})\boldsymbol{n}|_\Sigma; \\ T = A_T^{-1}\left(\boldsymbol{k}\boldsymbol{V}_\phi\right) + A_T^{-1}\boldsymbol{k}\left(A_3\left(\boldsymbol{T}(\boldsymbol{V})\boldsymbol{n}\right)_\Sigma\right) + \lambda A_T^{-1}T. \end{cases} \tag{11}$$

Σ_0 bilen $\boldsymbol{V}(\boldsymbol{x})$ funksiýanyň Σ çäkde yz alma operatoryny belgiläliň. Onda (10) deňligiň ikinjisinden

$$\Sigma_0 \boldsymbol{V} = \hat{\Sigma}\boldsymbol{u}$$

deňligi alarys. Bu ýerde $\hat{\Sigma} : \boldsymbol{W}_2^2(\Omega_0) \to \boldsymbol{W}_2^{\alpha-1/2}(\Sigma)$ yz alma operatory $\alpha > \frac{1}{2}$.

(11) sistemanyň ikinji deňlemesine Σ_0 operatoryny ulanalyň:

$$\Sigma_0 \boldsymbol{V} = \hat{\Sigma}\boldsymbol{u} = (\lambda+1)\Sigma_0 A_2^{-1}\boldsymbol{V} + \Sigma_0 A_2^{-1}\overline{P}\boldsymbol{k}T + \Sigma_0 A_3\left(T(\boldsymbol{V})\boldsymbol{n}|_\Sigma\right).$$

Onda soňky deňlemeden

$$T(\boldsymbol{V})\boldsymbol{n}|_\Sigma = -(\lambda+1)C_0^{-1}\Sigma_0 A_2^{-1}\boldsymbol{V} + C_0^{-1}\hat{\Sigma}\boldsymbol{u} - C_0^{-1}\Sigma_0 A_2^{-1}\overline{P}\boldsymbol{k}T \tag{12}$$

deňligi alarys. Bu ýerde $C_0 = \Sigma_0 A_3$. (10) deňlikleriň birinji şertini we (12) formulany ulanyp, (11) sistemadan alarys:

$$\begin{cases} \boldsymbol{u} = -\lambda^2 \rho A^{-1}\boldsymbol{u} + A^{-1}\boldsymbol{u} + (\lambda+1)A_1 C_0^{-1}\Sigma_0 A_2^{-1}\boldsymbol{V} - A_1 C_0^{-1}\hat{\Sigma}\boldsymbol{u} + A_1 C_0^{-1}\Sigma_0 A_2^{-1}\overline{P}\boldsymbol{k}T; \\ \boldsymbol{V} = (\lambda+1)A_2^{-1}\boldsymbol{V} + A_2^{-1}\overline{P}\boldsymbol{k}T - (\lambda+1)A_3 C_0^{-1}\Sigma_0 A_2^{-1}\boldsymbol{V} + A_3 C_0^{-1}\hat{\Sigma}\boldsymbol{u} - A_3 C_0^{-1}\Sigma_0 A_2^{-1}\overline{P}\boldsymbol{k}T; \\ T = A_T^{-1}(\boldsymbol{k}\boldsymbol{V}) - (\lambda+1)\boldsymbol{k}A_T^{-1}A_3 C_0^{-1}\Sigma_0 A_2^{-1}\boldsymbol{V} + \boldsymbol{k}A_T^{-1}A_3 C_0^{-1}\hat{\Sigma}\boldsymbol{u} - \boldsymbol{k}A_T^{-1}A_3 C_0^{-1}\Sigma_0 A_2^{-1}\overline{P}\boldsymbol{k}T; \end{cases}$$

$$\boldsymbol{u} = A^{-1/2}\xi;\ \boldsymbol{V} = A_2^{-1/2}\zeta;\ T = A_T^{-1/2}\eta. \tag{13}$$

ornuna goýmalary edeliň.

Soňky deňlemelere, degişlilikde, $A^{1/2}$, $A_2^{1/2}$ we $A_T^{1/2}$ operatorlary bilen täsir edip we

$$H = C_0^{-1/2}\hat{\Sigma}A^{-1/2};\ H^* = A^{1/2}A_1 C_0^{-1/2};\ H_0 = C^{-1/2}\Sigma_0 A_2^{-1/2};\ H_0^* = A_2^{1/2}A_3 C_0^{-1/2};\ B = A_2^{-1/2}\overline{P}\boldsymbol{k}A_T^{-1/2};$$

$$Q = H^*H;\ A_*^{-1} = A_2^{-1/2}\left(I - H_0^* H_0\right)A_2^{-1/2};\ G = A_T^{1/2}\boldsymbol{k}A_T^{-1}A_3 C_0^{-1/2} \tag{14}$$

belgilemeleri girizip, soňky sistemadan

$$\begin{cases} \left(I + Q - A^{-1} + \lambda^2 \rho A^{-1}\right)\xi - (\lambda+1)H^* H_0 A_2^{-1}\zeta - H^* H_0 B\eta = 0; \\ \left[A_2^{1/2}\left(I - (\lambda+1)A_*^{-1}\right)A_2^{-1/2}\right]\zeta - H_0^* H\xi + H^* H_0 B\eta = 0; \\ (I + GH_0 B)\eta - GH\xi - \left[A_T^{-1/2}\boldsymbol{k}A_2^{-1/2} - (\lambda+1)GH_0 A_2^{-1}\right]\zeta = 0 \end{cases}$$

sistemany almak bolar.

Bu sistemany

$$L(\lambda)\boldsymbol{Z}=0,\ \boldsymbol{Z}=\begin{pmatrix}\xi\\ \zeta\\ \eta\end{pmatrix} \tag{15}$$

görnüşde ýazmak bolar. Bu ýerde

$$L(\lambda)=\begin{pmatrix}I&0&0\\0&I&0\\0&0&I\end{pmatrix}-\begin{pmatrix}A^{-1}-Q & H^*H_0A_2^{-1} & H^*H_0B\\ H_0^*H & A_2^{1/2}A_*^{-1}A_2^{-1/2} & H_0^*H_0B\\ GH & GHA_2^{-1}-A_T^{-1/2}\boldsymbol{k}A_2^{-1/2} & GH_0B\end{pmatrix}-$$
$$-\lambda\begin{pmatrix}0 & H^*H_0A_2^{-1} & 0\\ 0 & A_2^{1/2}A_* & 0\\ 0 & GH_0A_2^{-1} & 0\end{pmatrix}+\lambda^2\begin{pmatrix}\rho A^{-1}&0&0\\0&0&0\\0&0&0\end{pmatrix}. \tag{16}$$

Şeýlelikde, aşakdaky lemma subut edildi:

3-nji lemma. (6)-(10) spektral mesele $\boldsymbol{L}_2(\Omega)\oplus\Im(\Omega_0)\oplus\boldsymbol{L}_2(\Omega_0)$ giňişlikde (15) deňlemä getirilýär. Bu ýerde $L(\lambda)$ operator-funksiýa (16) deňlikden kesgitlenilýär.

1-nji teorema. (16) deňlikden kesgitlenilýän $L(\lambda)$ operatorlar dessesiniň ((6)-(10) meseleleriň hem spektri ∞-de ýeke-täk predel nokady bolan tükenikli algebraik kratnyly hususy bahalardan ybarat bolup, hakyky oka simmetrik ýerleşendir. Şunlukda, hemme λ_k hususy bahalaryň tükenikli sanysyndan özgesi položitel ýarymoka we hyýaly oka ýanaşýan ýeterlikçe kiçi burçlara düşýärler.

Subudy. $\forall\ f_1\in\boldsymbol{L}_2(\Omega),\ \ \tilde f=(f_0,g_0,\psi_0)\in\boldsymbol{L}_2(\Omega)\oplus\Im(\Omega_0)\oplus\boldsymbol{L}_2(\Omega_0)$ wektorlar üçin

$$\hat\xi(\lambda)=\begin{pmatrix}\xi(\lambda)\\ \zeta(\lambda)\\ \eta(\lambda)\end{pmatrix}=\left[L^*(\lambda)\right]^{-1}\left\{\lambda\begin{pmatrix}f_1\\0\\0\end{pmatrix}+\begin{pmatrix}f_0\\g_0\\\psi_0\end{pmatrix}\right\} \tag{17}$$

wektor-funksiýa garalyň we onuň

$\Lambda_{\varepsilon,R}=\left\{\lambda\in C,\ \varepsilon<|\arg\lambda|<\dfrac{\pi}{2}-\varepsilon,\ |\arg\lambda|>\dfrac{\pi}{2}+\varepsilon,\ |\lambda|>R\right\}$ oblastda analitikdigini görkezeliň.

Bu ýerde:

$-\pi<\arg\lambda\le\pi,\ \ \varepsilon>0$ – erkin ýeterlikçe kiçi san;

$R=R(\varepsilon)$ – ýeterlikçe uly san saýlanyp alynýar.

$L^*(\lambda)$ operator-funksiýa bolsa $L(\lambda)$ operator-funksiýa çatrymlanandyr. (17) deňligi sistema görnüşinde ýazalyň, ýagny:

$$\begin{aligned}&\xi(\lambda)-\left(A^{-1}-Q\right)\xi(\lambda)-H^*H_0\zeta(\lambda)-H^*G^*\eta(\lambda)+\lambda^2\rho A^{-1}\xi(\lambda)=\lambda f_1+f_0;\\&\zeta(\lambda)-A_2^{-1}H_0^*H\xi(\lambda)-A_2^{-1/2}A_*^{-1}A_2^{1/2}\zeta(\lambda)-A_2^{-1}H^*G^*\eta(\lambda)+A_2^{-1/2}\boldsymbol{k}A_T^{-1/2}\eta(\lambda)-\\&-\lambda A_2^{-1}H_0^*H\xi(\lambda)-\lambda A_*^{-1}A_2^{-1/2}\zeta(\lambda)-\lambda A_2^{-1}H^*G^*\eta(\lambda)=g_0;\\&\eta(\lambda)-B^*H_0^*H\xi(\lambda)-B^*H_0^*H_0\zeta(\lambda)-B^*H_0^*G^*\eta(\lambda)=\psi_0.\end{aligned} \tag{18}$$

ýa-da

$$\left(I - A^{-1} + Q + \lambda^2 \rho A^{-1}\right)\xi(\lambda) - H^* H_0 \zeta(\lambda) - H^* G^* \eta(\lambda) = \lambda f_1 + f_0;$$
$$\left(I - A_2^{-1/2} A_*^{-1} A_2^{-1/2} - \lambda A_*^{-1} A_2^{-1/2}\right)\zeta(\lambda) - \left(A_2^{-1} H_0^* H + \lambda A_2^{-1} H_0^* H\right)\xi(\lambda) -$$
$$-\left(A_2^{-1} H^* G^* - A_2^{-1/2} \bar{k} A_T^{-1/2} + \lambda A_2^{-1} H^* G^*\right)\eta(\lambda) = g_0. \qquad (19)$$

$$\left(I - B^* H_0^* G^*\right)\eta(\lambda) - B^* H_0^* H \xi(\lambda) - B^* H_0^* H_0 \zeta(\lambda) = \psi_0. \qquad (20)$$

(19) deňlemeden $\lambda \in \Lambda_{\varepsilon,R}$ bolanda

$$\zeta(\lambda) = T_1(\lambda)\left(A_2^{-1} H_0^* H + \lambda A_2^{-1} H_0^* H\right)\xi(\lambda) +$$
$$+T_1(\lambda)\left(A_2^{-1} H^* G^* - A_2^{-1/2} \bar{k} A_T^{-1/2} + \lambda A_2^{-1} H^* G^*\right)\eta(\lambda) + T_1(\lambda) g_0 \qquad (21)$$

deňligi alarys. Bu ýerde

$$T_1(\lambda) = \left[I - \lambda A_*^{-1} A_2^{-1/2} - A_2^{-1/2} A_*^{-1} A_2^{-1/2}\right]^{-1}. \qquad (22)$$

$T_1(\lambda)$ operator-funksiýany

$$T_1(\lambda) = \left[I - T_2(\lambda)\right]^{-1}\left(I - \lambda A_*^{-1} A_2^{-1/2}\right)^{-1} \qquad (23)$$

görnüşde aňladalyň. Bu ýerde $T_2(\lambda) = \left(I - \lambda A_*^{-1} A_2^{-1/2}\right)^{-1} A_2^{-1/2} A_*^{-1} A_2^{-1/2}$, onda bu ýerden $\lambda \to \infty,\ \lambda \in \Lambda_{\varepsilon,R}$ bolanda, $T_1(\lambda) \to 0$ gelip çykýar. Diýmek, $\Lambda_{\varepsilon,R}$ oblastynda

$$\left\|T_1(\lambda)\right\| \le K_\varepsilon \qquad (24)$$

bahalandyrma adalatlydyr.

(21) deňligi (20) deňlikde goýup alarys:

$$\left(I - B^* H_0^* G^*\right)\eta(\lambda) - B^* H_0^* H \xi(\lambda) - B^* H_0^* H_0 T_1(\lambda)(\lambda + 1) A_2^{-1} H_0^* H \xi(\lambda) -$$
$$-B^* H_0^* H_0 T_1(\lambda)\left[(\lambda + 1) A_2^{-1} H^* G^* - A_2^{-1/2} \boldsymbol{k} A_T^{-1/2}\right] = B^* H_0^* H_0 T_1(\lambda) g_0 + \psi_0.$$

Bu deňlemeden

$$\eta(\lambda) = T_3(\lambda)\left[B^* H_0^* H + (\lambda + 1) B^* H_0^* H_0 T_1(\lambda) A_2^{-1} H_0^* H\right]\xi(\lambda) +$$
$$+T_3(\lambda) B^* H_0^* H_0 T_1(\lambda) g_0 + T_3(\lambda)\psi_0$$

ýa-da

$$\eta(\lambda) = T_3(\lambda)\left[B^* H_0 C_0^{-1/2} \hat{\Sigma} A^{-1/4+\delta} + (\lambda + 1) B^* H_0^* H_0 T_1(\lambda) A_2^{-1} H_0^* C^{-1/2} \hat{\Sigma} A^{-1/4+\delta}\right] A^{-1/4-\delta}\xi(\lambda) +$$
$$+T_3(\lambda) B^* H_0^* H_0 T_1(\lambda) g_0 + T_3(\lambda)\psi_0 \equiv G_1'(\lambda) A^{-1/4-\delta}\xi(\lambda) + D_1(\lambda) g_0 + T_3(\lambda)\psi_0 \qquad (25)$$

gelip çykýar.

Bu ýerde

$$G_1(\lambda) = T_3(\lambda)\left[B^* H_0 C_0^{-1/2} \hat{\Sigma} A^{-1/4+\delta} + (\lambda + 1) B^* H_0^* H_0 T_1(\lambda) A_2^{-1} H_0^* C^{-1/2} \hat{\Sigma} A^{-1/4+\delta}\right]$$

$$D_1(\lambda) = T_3(\lambda) B^* H_0^* H_0 T_1(\lambda) g_0;$$

$$T_3(\lambda) = \left[I - (\lambda + 1) B^* H_0^* H_0 T_1(\lambda) A_2^{-1} H^* G^* + B^* H_0^* H_0 T_1(\lambda) A_2^{-1} \boldsymbol{k} A_T^{-1/2} - B^* H_0^* G^*\right]^{-1};\ \ 0 < \delta < \frac{1}{4}.$$

(24) bahalandyrmadan peýdalanyp $\Lambda_{\varepsilon,R}$ oblastda

$$\left\|T_3(\lambda)\right\| \le K_\varepsilon \tag{26}$$

bahalandyrmanyň ýerine ýetýändigini görmek bolýar. (25) deňligi (21) deňlikde ornuna goýup alarys:

$$\zeta(\lambda) = (\lambda+1)T_1(\lambda)A_2^{-1}H_0^*H\xi(\lambda) + T_1(\lambda)\left[(\lambda+1)A_2^{-1}H^*G^* - A_2^{-1/2}\boldsymbol{k}A_T^{-1/2}\right]\times$$
$$\times\left[G_1(\lambda)A^{-1/4-\delta}\xi(\lambda) + D_1(\lambda)g_0 + T_1(\lambda)\psi_0\right] + T_1(\lambda)g_0$$

ýa-da

$$\zeta(\lambda) = G_2(\lambda)A^{-1/4-\delta}\xi(\lambda) + D_2(\lambda)g_0 + D_3(\lambda)\psi_0. \tag{27}$$

Bu ýerde

$$G_2(\lambda) = (\lambda+1)T_1(\lambda)A_2^{-1}H^*G^*G_1(\lambda) + (\lambda+1)T_1(\lambda)A_2^{-1}H_0^*C^{-1/2}\hat{\Sigma}A^{-1/4+\delta} =$$
$$= (\lambda+1)T_1(\lambda)A_2^{-1}\left[H^*G^*G_1(\lambda) + H_0^*C^{-1/2}\hat{\Sigma}A^{-1/4+\delta}\right];$$
$$D_2(\lambda) = T_1(\lambda) + T_1(\lambda)\left[(\lambda+1)A_2^{-1}H^*G^* - A_2^{-1/2}\boldsymbol{k}A_T^{-1/2}\right]D_1(\lambda);$$

$$D_3(\lambda) = T_1(\lambda)\left[(\lambda+1)A_2^{-1}H^*G^* - A_2^{-1/2}\boldsymbol{k}A_T^{-1/2}\right]T_3(\lambda).$$

Indi $\zeta(\lambda)$ we $\eta(\lambda)$ wektor-funksiýalar üçin tapylan aňlatmalary (25) we (27) deňliklerden (18) deňlikde goýalyň. Onda alarys:

$$\left(I - A^{-1} + Q + \lambda^2\rho A^{-1}\right)\xi(\lambda) - H^*H_0G_2(\lambda)A^{-1/4-\delta}\xi(\lambda) - H^*H_0D_2(\lambda)g_0 - H^*H_0D_3(\lambda)\psi_0 -$$
$$-H^*G^*G_1(\lambda)A^{-1/4-\delta}\xi(\lambda) - H^*G^*D_1(\lambda)g_0 - H^*G^*T_3(\lambda)\psi_0 = \lambda f_1 + f_0.$$

Bu deňligi

$$l(\lambda)\xi(\lambda) \equiv \left[I + Q - A^{-1} + \lambda^2\rho A^{-1} - H^*H_0G_2(\lambda)A^{-1/4-\delta} - H^*G^*G_1(\lambda)A^{-1/4-\delta}\right]\xi(\lambda) = f(\lambda)$$

görnüşde ýazmak bolar. Bu ýerde

$$l(\lambda) = I + Q - A^{-1} + \lambda^2\rho A^{-1} - H^*H_0G_2(\lambda)A^{-1/4-\delta} - H^*G^*G_1(\lambda)A^{-1/4-\delta}. \tag{28}$$

$$f(\lambda) = H^*H_0D_2(\lambda)g_0 + H^*G^*D_1(\lambda)g_0 + H^*H_0D_3(\lambda)\psi_0 + H^*G^*T_3(\lambda)\psi_0 + \lambda f_1 + f_0. \tag{29}$$

Şeýlelikde, $\Lambda_{\varepsilon,R}$ oblastynda

$$l(\lambda)\xi(\lambda) = f(\lambda) \tag{30}$$

deňlik adalatlydyr. Bu ýerde $l(\lambda)$ operator-funksiýa $f(\lambda)$ wektor-funksiýa $\Lambda_{\varepsilon,R}$ oblastynda analitikdir.

(M.B. Orazow, 1981; M.B. Orazow, 1982) işleriň netijelerinden

$$A_*^{-1} = A_2^{-1/2}\left(I - H_0^*H_0\right)A_2^{-1/2}$$

üçin $KerA_*^{-1} = 0,\ D\left(A_*^{1/4}\right) = D\left(A_2^{1/4}\right) = \widetilde{\boldsymbol{W}}_2^{1/2}(\Omega_0)$ gelip çykýar. Onda $H,\ H^*,\ H_0,\ H_0^*,\ B,\ Q,$ G operatorlarynyň kesgitlenilişinden we häsiýetlerinden peýdalanyp, (M.B. Orazow, 1981; M.B. Orazow, 1982; T.B. Radziýewskiý, 1976) işlere laýyklykda $\forall\delta > 0$ san üçin $\Lambda_{\varepsilon,R}$ oblastynda

$$\left\|G_1(\lambda)\right\| = o\left(|\lambda|^\delta\right),\ \left\|G_2(\lambda)\right\| = o\left(|\lambda|^\delta\right) \tag{31}$$

bahalandyrmalary almak bolýar. Edil şulara meňzeşlikde $\lambda \in \Lambda_{\varepsilon,R},\ \lambda \to \infty$ bolanda

$$\left\|D_1(\lambda)g_0\right\| = o(1),\ \left\|D_2(\lambda)g_0\right\| = o(1),\ \left\|D(\lambda)\psi_0\right\| = o(1),\ \left\|T_3(\lambda)\psi_0\right\| = o(1) \quad (32)$$

bahalandyrmalardan $\lambda \to \infty$ ymtylanda

$$\left\|f(\lambda)\right\| \le |\lambda|\left\|f_1\right\| + o(1) \quad (33)$$

bahalandyrmanyň ýerine ýetýändigini görmek bolýar.

4-nji lemma. $R = R(\varepsilon)$ ýeterlikçe uly bolanda $\Lambda_{\varepsilon,R}$ oblastynda (28) deňlikden kesgitlenilýän $l(\lambda)$ operator-funksiýa üçin

$$\left\|l^{-1}(\lambda)\right\| \le K_\varepsilon \quad (34)$$

bahalandyrma adalatlydyr.

Subudy. $l(\lambda)$ operator-funksiýany

$$l(\lambda) = \left(I - i\rho^{1/2}\lambda A^{-1/2}\right)\left[I + T_4(\lambda)\right]\left(I + i\rho^{1/2}\lambda A^{-1/2}\right) \quad (35)$$

görnüşde ýazalyň. Bu ýerde

$$T_4(\lambda) = \left(I - i\rho^{1/2}\lambda A^{-1/2}\right)^{-1}\left[Q - H^* H_0 G_2(\lambda) A^{-1/4-\delta} - H^* G^* G_1(\lambda) A^{-1/4-\delta}\right]\left(I + i\rho^{1/2}\lambda A^{-1/2}\right)^{-1}$$

(31) bahalandyrmalary ulanyp, $\lambda \in \Lambda_{\varepsilon,R}$, $\lambda \to \infty$ bolanda (T.B. Radziýewskiý, 1976) işe laýyklykda $\left\|T_4(\lambda)\right\| \to 0$ bolýandygy gelip çykýar. Onda $\lambda \in \Lambda_{\varepsilon,R}$, $\lambda \to \infty$ bolanda

$$l^{-1}(\lambda) = \left(I - i\rho^{1/2}\lambda A^{-1/2}\right)^{-1}\left[I + T_4(\lambda)\right]^{-1}\left(I + i\rho^{1/2}\lambda A^{-1/2}\right)^{-1} \quad (36)$$

operator-funksiýa bardyr we (34) bahalandyrma adalatlydyr. Lemma subut edildi.

$l(\lambda)\xi(\lambda) = f(\lambda)$ we (36) deňliklerden

$$\xi(\lambda) = l^{-1}(\lambda) f(\lambda),\ \lambda \in \Lambda_{\varepsilon,R} \quad (37)$$

deňligi alarys. Bu ýerden ýeterlikçe uly λ üçin $\xi(\lambda)$ wektor-funksiýanyň analitikdigi gelip çykýar. (31)-(33) bahalandyrmalardan

$$\left\|\xi(\lambda)\right\| = o(|\lambda|),\ \lambda \to \infty,\ \lambda \in \Lambda_{\varepsilon,R} \quad (38)$$

deňligi alarys. Onda (25)-(27) deňliklerden (36), (38) deňlikleri göz öňünde tutup,

$$\left\|\zeta(\lambda)\right\| = o(|\lambda|),\ \left\|\eta(\lambda)\right\| = o(|\lambda|),\ \lambda \in \Lambda_{\varepsilon,R} \quad (39)$$

bahalandyrmalary, şeýle-de $\Lambda_{\varepsilon,R}$ oblastynda $\zeta(\lambda)$ we $\eta(\lambda)$ wektor-funksiýalaryň analitikdigini alarys.

$f_1 \in \boldsymbol{L}_2(\Omega)$, $(f_0, g_0, \psi_0) \in \boldsymbol{L}_2(\Omega) \oplus \Im(\Omega_0) \oplus \boldsymbol{L}_2(\Omega_0)$ wektorlaryň erkin saýlanyp alnandygyny göz öňünde tutsak, R ýeterlikçe uly bolanda $\Lambda_{\varepsilon,R}$ oblastynda $\hat{\xi}(\lambda)$ wektor-funksiýanyň analitikdigi gelip çykýar. Diýmek, bu oblastda $L(\lambda)$ operator-funksiýanyň hususy bahalary ýokdur. $L(\lambda)$ operatorlar dessesiniň we spektral meseläniň spektriniň ýeke-täk predel nokady tükeniksizlikde bolan tükenikli kratnyly λ_k hususy bahalardan durýandygy (16) operatorlar dessesine girýän operatorlaryň häsiýetlerinden we $L(\lambda)$ dessesiniň öz-özüne çatrymlanmadyk kwadratik desse bolýandygyndan gelip çykýar. Teorema subut edildi.

Umumylygy kemeltmezden ýönekeýlik üçin hemme λ_k hususy bahalary ýönekeý diýip hasap edeliň. Goý, (ξ_k, ζ_k, η_k) wektor $L(\lambda)$ operatorlar dessesiniň λ_k hususy bahalara degişli hususy wektorlary bolsun. $\Phi_k = (\xi_k, \zeta_k, \eta_k, \lambda_k \xi_k)$ wektora garalyň.

2-nji teorema. Hemme $\{\Phi_k\}$ wektorlar sistemasy $\boldsymbol{L}_2(\Omega)\oplus\Im(\Omega_0)\oplus\boldsymbol{L}_2(\Omega_0)\oplus\boldsymbol{L}_2(\Omega)$ giňişliginde doly we minimaldyr.

Subudy. Goý, f_1 we (f_0, g_0, ψ_0) wektorlar (17) deňlikden kesgitlenilýän $\hat{\xi}(\lambda)$ wektor-funksiýanyň bitinligini üpjün edýän bolsun. Onda ýeterlikçe kiçi $\varepsilon > 0$ sany saýlap alyp, we $\hat{\xi}(\lambda)$ wektor-funksiýanyň tükenikli ösüşe eýedigini göz öňünde tutup, bu funksiýa $\Lambda_{\varepsilon,R}$ oblastynda Fragmen-Lindelefiň teoremasyny ulanyp (A.I. Markuşewiç, 2006), $\Lambda_{\varepsilon,R}$ oblastynda (38)-(39) bahalandyrmalardan peýdalanyp

$$\xi(\lambda)=\xi_0,\ \zeta(\lambda)=\zeta_0,\ \eta(\lambda)=\eta_0$$

bolýandygyny görmek bolýar. Onda (18)-(20) deňliklere gaýdyp gelip, olardan $\xi_0=0,\ \zeta_0=0,\ \eta_0=0$ gelip çykýandygyna göz ýetireris. Diýmek, $f_1=0,\ f_0=0,\ g_0=0,\ \psi_0=0.$ Bu bolsa garalýan wektorlar sistemasynyň dolulygyny aňladýar. Indi $\{\Phi_k\}$ sistemanyň minimallygyny görkezeliň. Onuň üçin (18)-(20) sistemany $\xi_1=\xi,\ \xi_2=\zeta,\ \xi_3=\eta,\ \xi_4=\lambda\xi$ ornuna goýmalaryň kömegi bilen çyzykly operatorlar dessesine getirip bolýandygyny bellemek ýeterlikdir (I.S. Gohberg, 1965). Teorema subut edildi.

Bellik. (6)-(10) meseläniň λ_k hususy bahalara degişli hususy wektorlarynyň dolulygy baradaky tassyklamany almak üçin (15) deňlemeden (6)-(10) meselä gaýdyp gelmegi yzarlamak ýeterlikdir.

EDEBIÝAT

1. *Aşirow A.* Konweksiýaly suwuklygyň yrgyldylary baradaky mesele dogrusynda. Berkarar döwletiň täze eýýamynyň Galkynyşy döwründe ylym-bilim ulgamy ösüş ýolunda (tezisler ýygyndysy). – Aşgabat: Ylym, 2022. – S. 244-246.
2. *Aşirow A.* Gapdaky konweksiýaly suwuklygyň yrgyldylary baradaky mesele dogrusynda. «Berkarar döwletiň täze eýýamynyň Galkynyşy döwri: tebigy we takyk ylymlar ulgamynyň kämilleşdirilmegi ýurdumyzyň durnukly ösüşiniň berk binýadydyr» atly ylmy-amaly maslahatyň nutuklarynyň gysgaça beýany. – Aşgabat: Ylym, 2022. – S. 164-166.
3. *Aşirow A*. Erkin konweksiýalar nazaryýetiniň bir meselesi barada. // Berkarar döwletimiziň bagtyýarlyk döwrüniň ylmy gadamlary. Ylmy makalalaryň ýygyndysy. – A.: Ylym, – 2023. – S. 5-11.
4. *Гохберг И.Ц. Крейн М.Г.* Введение в теорию линейных несамосопряженных операторов в гильбертовом пространстве. – М.: Наука, 1965. – С. 448.
5. *Крейн С.Г.* К задаче о движении вязкой жидкости в открытом сосуде // Функциональный анализ и его приложения. – Т. 2, №1, – 1968.– С. 40-50.
6. *Маркушевич А.И.* Теория аналитических функций, – М.: Мир, 2006.
7. *Оразов М.Б.* К задаче о нормальных колебаниях вязкой жидкости, частично заполняющей цилиндрическую упругую оболочку. – Т. 259, – §1. // ДАН СССР. – 1981. – С. 79-82.
8. *Оразов М.Б.* К задаче о нормальных колебаниях вязкой жидкости, частично заполняющей упругий сосуд. // Функц. анализ и его прилож. – Т. I6, – №1, – 1982.
9. *Радзиевский Г.В*. Квадратичный пучок операторов. // Киев: Препринт, 1976.

ЗАДАЧА О НОРМАЛЬНЫХ КОЛЕБАНИЯХ ВЯЗКОЙ ЖИДКОСТИ С КОНВЕКЦИЕЙ

Аллаберды Аширов
Президент Академии наук Туркменистана

Аннотация

В этой статье были исследованы задачи о нормальных колебаниях несжимаемой вязкой жидкости с конвекцией в упругом сосуде. Доказаны теоремы о структур частной оценки соответствующей спектральной задачи и их расположение, а также о минимальности и полноты частных векторов пучка операторов.

Ключевые слова: упругий сосуд, спектральная задача, частная оценка, частная функция, несжимаемая жидкость, вязкая жидкость, конвекция, функциональное пространство, операторы, пучок операторов, полнота.

THE PROBLEM OF NORMAL OSCILLATIONS OF A VISCOUS FLUID WITH CONVECTION

Allaberdy Ashirov
The President of Academy of Science of Turkmenistan

Abstract

In this article, the problems of normal oscillations of an incompressible viscous fluid with convection in an elastic vessel were studied. Theorems are proved on the structures of a partial estimate of the corresponding spectral problem and their location, as well as on the minimality and completeness of the partial vectors of a bunch of operators.

Keywords: elastic vessel, spectral problem, partial estimate, partial function, incompressible fluid, viscous fluid, convection, function space, operators, a bunch of operators, completeness.